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Analytical Studies on Option Pricing Model

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Research Article

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ABSTRACT

The well-known model for an option pricing is the Black-Scholes model. To obtain analytical solutions of the Black-Scholes equation the combination of Lie group transformation and Chebyshev approximation is considered. But instead of the classical Black-Scholes equation in this work the parametric expansion of the Black-Scholes equation is considered. Due to the modification of the equation, the implementation of the Lie group symmetries is modified too.

1. Introduction

Assuming the value of an option $u(x,t)$at the time t, the considered semi-linear partial differential equation is the Black-Scholes equation for the price of the European call option

$$u_t + \frac{1}{2}\sigma^2 x^2 u_{xx} + rxu_x + f(u) = 0 \tag{1}$$

with the terminal condition

$$u(x,T) = \begin{cases} x-\kappa, x>\kappa \\ 0 \quad ,x\le\kappa \end{cases} \tag{2}$$

Where σ is the volatility of the stock, r is the risk-free interest rate, κ is the strike price and T is the time of the option. To solve Eq(1), in the literature using a series of transformations, the equation is reduced into a heat equation or nonlinear evolution equation. The reduced equation is solved by numerical methods or analytical methods.

For the Lie group symmetries generally, as a reduced equation the heat equation is considered [1-4] are considered symmetry breaking of the nonlinear Black-Scholes equation.

Therefore, as we mentioned above, the Black-Scholes equation and its one of the expansion are considered and their symmetry analysis is studied but as a result, the reduced equation is obtained as a well-known heat equation.

In this view, the Eq(1) is reconsidered by adding the parameter a, therefore the Eq(1) is converted into the parametric expansion of the Black-Scholes equation

$$au_t + \frac{1}{2}\sigma^2 x^2 u_{xx} + rxu_x + f(u) = 0 \tag{3}$$

With the parameter a , the dimension of the solution space is increased and also new symmetries are held [5,6]. By considering so, the space for Eq(1) is (x,t,u) whereas for Eq(3), as we mentioned, (x,t,u,a) The one-parameter Lie group of the infinitesimal transformations for the expanded space (x,t,u,a) is considered

$$X = \zeta_1 \frac{\partial}{\partial x} + \zeta_2 \frac{\partial}{\partial t} + \zeta_3 \frac{\partial}{\partial a} + \eta \frac{\partial}{\partial u} \tag{4}$$

where only ζ_3 is depended on only the parameter a, while ζ_1, ζ_2, η are depended on (x,t,u,a) and leaves Eq(3) invariant. Generally, 1st prolongation is considered to obtain infinitesimal generators but for Eq(3) 2nd prolongation is used

$$\begin{aligned} Pr^{(2)}X = \zeta_1 \frac{\partial}{\partial x} + \zeta_2 \frac{\partial}{\partial t} + \zeta_3 \frac{\partial}{\partial a} + \phi_1 \frac{\partial}{\partial u} + \zeta_{10} \frac{\partial}{\partial u_x} \\ + \zeta_{01} \frac{\partial}{\partial u_t} + \zeta_{20} \frac{\partial}{\partial u_{xx}} \end{aligned} \tag{5}$$

Applying the procedure of the Lie symmetries [6,7,8], with the determining equations, the symmetry generator is obtained

$$\begin{aligned} X = \left(-\frac{1}{6}c_1 x^2 e^{\frac{-rt}{a}} + \frac{1}{x}c_3\right)\frac{\partial}{\partial x} + \left(c_4 e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial t} \\ + \left(-ac_1 x e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial a} \\ + \left(x e^{\frac{-rt}{a}}(c_1 u + c_2)\right)\frac{\partial}{\partial u} \end{aligned} \tag{6}$$

Now, with respect to c_i, the basis vectors are obtained;

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$$
\begin{aligned}
c_1 = 1: X_1 &= \left(-\frac{1}{6}x^2 e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial x} + \left(-axe^{\frac{-rt}{a}}\right)\frac{\partial}{\partial a} \\
&\quad + \left(xe^{\frac{-rt}{a}}u\right)\frac{\partial}{\partial u} \\
c_2 = 1: X_2 &= \left(xe^{\frac{-rt}{a}}\right)\frac{\partial}{\partial u} \\
c_3 = 1: X_3 &= \left(\frac{1}{x}\right)\frac{\partial}{\partial x} \\
c_4 = 1: X_4 &= \left(e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial t}
\end{aligned} \tag{7}
$$

The corresponding commutator table is given as follows:

Table 1. Commutator table of basis vectors.

$[,]$	X_1	X_2	X_3	X_4
X_1	0	$X_2 + X_1$	X_1	$X_1 + X_4$
X_2	$-X_2 - X_1$	0	X_2	X_2
X_3	$-X_1$	$-X_2$	0	0
X_4	$-X_1 - X_4$	$-X_2$	0	0

As it is seen from the commutator table, new sub-algebras are obtained by the operators such as $X_1 + X_4$ and $X_2 + X_1$ Now, we will consider the reduced equations via these operators and their solutions with the appropriate methodology.

Case 1: One of the new operators is

$$
\begin{aligned}
X_2 + X_1 &= \left(-\frac{1}{6}x^2 e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial x} + \left(-axe^{\frac{-rt}{a}}\right)\frac{\partial}{\partial a} \\
&\quad + \left(xe^{\frac{-rt}{a}}(u+1)\right)\frac{\partial}{\partial u}
\end{aligned} \tag{8}
$$

And the corresponding similarity transformation is $u = \frac{v(z)}{a} - 1, z = \frac{x^6}{a}$, therefore the Eq. (3) is reduced into the following equation

$$18\sigma^2 z^2 v'' + (15\sigma^2 + 6B)zv' - rv + ra = 0 \tag{9}$$

Case 2: The new operator is

$$
\begin{aligned}
X_4 + X_1 &= \left(-\frac{1}{6}x^2 e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial x} + \left(-axe^{\frac{-rt}{a}}\right)\frac{\partial}{\partial a} \\
&\quad + \left(xe^{\frac{-rt}{a}}u\right)\frac{\partial}{\partial u} + \left(e^{\frac{-rt}{a}}\right)\frac{\partial}{\partial t}
\end{aligned} \tag{10}
$$

And the corresponding similarity transformation is $u = \frac{v(z)}{x^6}, z = \frac{a}{C-B}e^{(C-B)t/a} - \frac{6}{x}$, therefore the Eq. (3) is reduced into the following equation

$$
\begin{aligned}
18\frac{\sigma^2}{x^2}v'' &+ \left[a(7\sigma^2 + C)e^{(C-B)t/a} \right. \\
&\quad \left. - (7\sigma^2 + B)z\right]v' \\
&\quad + (21\sigma^2 - 6B - C)v = 0
\end{aligned} \tag{11}
$$

To obtain solutions for Eq(10) and Eq(11) is not possible directly, but several approximate methods are known in the literature [7-10]. They are used to solve the nonlinear evolution equations but now we use these methods to solve variable coefficient differential equations. Among these methods, the auxiliary equation method is a common one and many exact solutions of various auxiliary equations have been utilized [11-24]. The most common properties of some of these methods are based on the polynomial approximations in ∞ - norm. But, the choice of a norm can extensively manipulate the outcome of the problem. That is to say that the polynomial approximation to a given continuous function in one norm need not carry any resemblance to the polynomial approximation in another norm. As a result, the choice of the norm will be executed in the sense in which the given continuous function must be a "good" approximation for many practical problems. Therefore a "good" approximation in the 2-norm (L^2) is closely related to the concept of orthogonality and this, consequently leads to the concept of inner product and may lead to the solutions of specific orthogonal polynomials which form complete orthogonal sets in L^2.

In this manner, it is known that new applications of orthogonal polynomials from the higher-order polynomial classes and their connections to other areas of mathematics, physics, and engineering have emerged during the past few years. We consider the analogous problem of best approximation in the 2-norm and the best approximation in the 2-norm is closely related to the notion of orthogonality which forms a new solution space.

2. Chebyshev polynomials and Methodology

Chebyshev polynomials form a special class of polynomials that are orthogonal especially suited for approximating other functions. Chebyshev polynomials are polynomials with the largest possible leading coefficient, but subject to the condition that their absolute value on the interval [-1,1] is bounded by 1. They are also the extremal polynomials for many other properties [15]. They are widely used in many areas of numerical analysis: uniform approximation, least-squares approximation, numerical solution of ordinary and partial differential equations (the so-called spectral or pseudospectral methods), and so on. Because, it is one of the very few functions for which it is possible to write down in simple closed-form the minimax polynomial. One such problem of practical importance concerns the approximation of a power of x by a polynomial of lower degree. The minimax approximation, in this case, is given in terms of Chebyshev polynomials. In this approximation, we use the differential equation of Chebyshev polynomials,

$$(1-\zeta^2)y''(\zeta) - \zeta y'(\zeta) + n^2 y(\zeta) = 0 \tag{12}$$

with the transformation $\omega = \cos\zeta$, Eq(12) turns into

$$y''(\omega) + n^2 y(\omega) = 0 \tag{13}$$

Eq(10) is considered as the auxiliary equation to solve the nonlinear partial differential equation. For a large class of the equations of the nonlinear partial differential equations have exact solutions which can be constructed via finite series

$$u(\zeta) = \sum_{i=0}^{N} g_i T_n^i(\omega) \tag{14}$$

Here, $g_i, (i = 0,1,\dots,N)$ are parameters to be further determined, N is an integer fixed by a balancing principle and elementary function $T_n(\zeta)$, Chebyshev function, is the solution of Chebyshev equation referred to as the auxiliary equation [15,16,17,19,21,25].

It is worth pointing parameters with that we happen to know the general solution(s), of the auxiliary equation beforehand or we know at least exact analytical particular solutions of the auxiliary equation.

Substitute Eq.(14) into reduced differential equation to determine the parameters $g_i, (i = 0,1, \dots, N)$ with the aid of symbolic computation.

It is very apparent that determining the elementary function $T_n(\zeta)$, via auxiliary equation is crucial and plays a very important role finding new travelling wave solutions of nonlinear evolution equations. This fact, indeed, compels researchers to search for a novel auxiliary equations with exact solutions.

It is very apparent that if the elementary function, $T_n(\zeta)$, is an orthogonal function that forms complete the orthogonal sets in L^2 then, the solution series will be convergent series, therefore, the series (14) will converge rapidly.

In this study, we use Chebyshev polynomials in the solution series (14) to solve the reduced equations (Eq(10) and Eq(11)):

Case 1: We consider the reduced equation

$$18\sigma^2 z^2 v'' + (15\sigma^2 + 6B)zv' - rv + ra = 0 \quad (15)$$

via the operator $X_2 + X_1$ and the transformation is $u = \frac{v(z)}{a} - 1, z = \frac{x^6}{a}$ The reduced equation is a well-known equation with variable coefficients, Cauchy Euler equation. The exact solution is only space-dependent solution.

$$u(x,t) = \frac{x^6}{a^2} C_2 + \frac{C_1}{a}\left(\frac{x^6}{a}\right)^{\left(\frac{-r}{3\sigma^2}\right)} \quad (16)$$

In addition to the given solution, applying the given procedure, the solution sets of the nonlinear algebraic system are given below;

Substituting the parameters and the solution of the Chebyshev differential equation into the ansatz, the solution is obtained;

Solution set 1 :

$r = -36\sigma^2\zeta^2 n^2, c_1 = 0, C_1 = 0,$

$$c_0 = \frac{-36\sigma^4\zeta^2 n^2 {c_1}^2 + 100c_2\sigma^4 a + 4c_1 B^2 + 80c_2\sigma^2 Ba + 25\sigma^4 {c_1}^2 + 4c_2 B^2 a + 20\sigma^2 B {c_1}^2}{4c_2(25\sigma^4 + 20\sigma^2 B + 4B^2)} \quad (17)$$

$$u(x,t) = \frac{-36\sigma^4 \cos^2\left(\frac{x^6}{a}\right) n^2 {c_1}^2 + 100c_2\sigma^4 a + 4c_1 B^2 + 80c_2\sigma^2 Ba + 25\sigma^4 {c_1}^2 + 4c_2 B^2 a + 20\sigma^2 B {c_1}^2}{c_2(25\sigma^4 + 20\sigma^2 B + 4B^2)} \quad (18)$$
$$+ c_2 {C_2}^2 \cos^2\left(n \cos\left(\frac{x^6}{a}\right)\right)$$

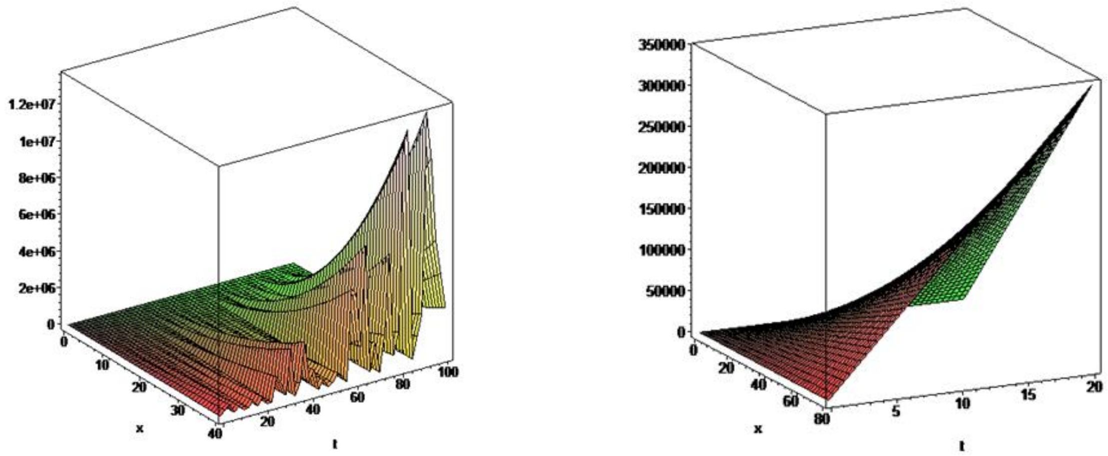


Fig. 1 The exact solution of the Eq(10) reduced by the $X_2 + X_1$ operator via Chebyshev approximation method and direct method, respectively.

The Figure of the solution is given for $a = 0.01, \sigma = 0.12, B = 0.1, C_2 = t, n = 2, c_2 = x$

Case 2: We consider the reduced equation

$$18\frac{\sigma^2}{x^2} v'' + \left[a(7\sigma^2 + C)e^{(C-B)t/a} - (7\sigma^2 + B)z\right]v' + (21\sigma^2 - 6B - C)v = 0 \quad (19)$$

via the operator $X_4 + X_1$ and the transformation is $u = \frac{v(z)}{x^6}, z = \frac{a}{C-B}e^{(C-B)t/a} - \frac{6}{x}$. Applying the given procedure, the solution sets of the nonlinear algebraic system are given below;

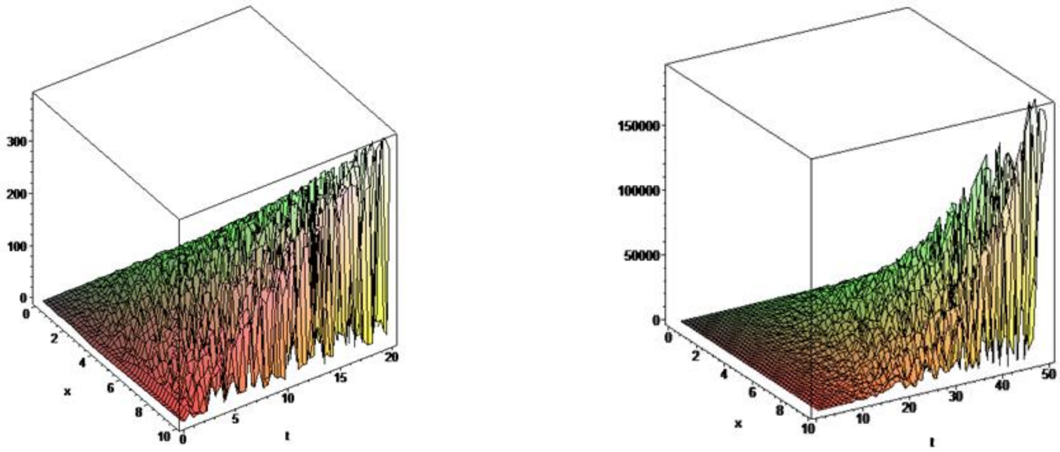
Solution set 1:

$$B = -\frac{-21x^2A^2 + x^2C + 18A^2n^2}{6x^2}, c_2 = 0 \quad (20)$$

Solution set 2:

$$B=-\frac{-21x^2A^2+x^2C+36A^2n^2}{6x^2}$$

$$c_0=\frac{\frac{1}{4}{c_1}^2\left(\begin{array}{l}14112x^4A^2C^3-45864x^4A^2C^2-296352x^4A^2C-432x^4A^2C^2\zeta^2n^2-2592x^3A^2C^2\zeta n^2\\+504x^6A^2C\zeta^2-924x^6A^2C^2\zeta^2+5544x^5A^2C^2\zeta+2286144A^8n^4+259308x^5A^8\zeta\\+12348x^6A^6\zeta^2+74088x^5A^6\zeta+653184n^4A^6C+46656n^4A^4C^2+21609x^6A^8\zeta^2\\-2667168x^2A^8n^2-588x^6A^4C\zeta^2-1274x^6A^4C^2\zeta^2-15288x^5A^4C^2\zeta-98784x^5A^6C\zeta\\-3528x^5A^4C\zeta+63504x^2A^8\zeta^2n^4-74088x^4n^2A^8\zeta^2+762048xA^8n^4\zeta-889056x^3n^2A^8\zeta\\-21168x^4A^6n^2\zeta^2-127008x^3A^6n^2\zeta+4704x^5A^2C^3\zeta+18144x^2A^2C^3n^2+199584x^2A^4C^2n^2\\+127008x^2A^6Cn^2-8232x^6A^6C\zeta^2+392x^6A^2C^3\zeta^2+5544x^4A^4C^2\zeta^2n^2+66528x^3A^4C^2\zeta n^2\\+1296x^2A^4C^2\zeta^2n^4+15552xA^4C^2n^4\zeta+504x^4A^2C^3n^2\zeta^2+6048x^3A^2C^3n^2\zeta+18144x^2A^6Cn^4\zeta^2\\+3528x^4A^6Cn^2\zeta^2+217728xA^6Cn^4\zeta+42336x^3A^6Cn^2\zeta-6048x^4A^4Cn^2\zeta^2-36288x^3A^4Cn^2\zeta\\1764x^6A^4\zeta^2+49x^6\zeta^2C^4-84x^6\zeta^2C^3+588x^5C^4\zeta+36x^6C^2\zeta^2-504x^5C^3\zeta-11664x^2A^4n^2\\+777924x^4A^8+1764C^4x^4\end{array}\right)}{c_2\left(\begin{array}{l}14112x^4A^2C^3-45864x^4A^4C^2-296352x^4A^6C-432x^4A^2C^2\zeta^2n^2-2592x^3A^2C^2n^2\zeta\\+504x^6A^2C\zeta^2-924x^6A^2C^2\zeta^2-5544x^5A^2C^2\zeta+2286144n^4A^8+259308x^5A^8\zeta\\+12348x^6A^6\zeta^2+74088x^5A^6\zeta+653184n^4A^6C+46656n^4A^4C^2+21609x^6A^8\zeta^2\\-2667168x^2A^8n^2-588x^6A^4C\zeta^2-1274x^6A^4C^2\zeta^2-15288x^5A^4C^2\zeta-98784x^5A^6C\zeta\\-3528x^5A^4C\zeta+63504x^2A^8n^4\zeta^2-74088x^4A^8n^2\zeta^2+762048n^4A^8x\zeta-889056x^3A^8n^2\zeta\\-21168x^4A^6n^2\zeta^2-127008x^3A^6n^2\zeta+4704x^5A^2C^3\zeta+18144x^2A^2C^3n^2+199584x^2A^4C^2n^2\\127008x^2A^6Cn^2-8232x^6A^6C\zeta^2+392x^6A^2C^3\zeta^2+5544x^4A^2C^2n^2\zeta^2+66528x^3A^4C^2n^2\zeta\\+1296x^2A^4C^2n^4\zeta^2+15552n^4A^4C^2x\zeta+504x^4A^2C^3n^2\zeta^2+6048x^3A^2C^3n^3\zeta+18144x^2A^6n^4C\zeta^2\\+3528x^4A^6Cn^2\zeta^2+217728xA^6Cn^4\zeta+42336x^3A^6Cn^2\zeta-6048x^4A^4Cn^2\zeta^2-36288x^3A^4Cn^2\zeta\\+1764x^6A^4\zeta^2+49x^6C^4\zeta^2-84x^6C^3\zeta^2+588x^5C^4\zeta+36x^6C^2\zeta^2-504x^5C^3\zeta\\+777924x^4A^8+1764C^4x^4\end{array}\right)} \tag{21}$$



(1) (2)

Fig. 2 The exact solution of the Eq(11) reduced by the X_4+X_1 operator.

Substituting the parameters and the solution of the Chebyshev differential equation into the ansatz, the solutions are given in Fig. 2.

3. Conclusion

As seen that the classical Lie group transformation is not enough to solve nonlinear and semi-linear partial differential equations. Because of this reason, adding the parameter a , the dimension of the solution space is increased and also new symmetries are held via 2^{nd} prolongation. From the commutator table, we consider the sub-algebras which reduce the equation's non-solvable form. To solve reduced equations, we consider that one of the variants of the auxiliary equation method is known as the Cheybshev approximation method. Hence, the analytical solutions of the Black-Scholes equation are obtained by the combination of expanded Lie group transformations and the Chebyshev approximation method. However, to our knowledge, these methodologies were not studied in the literature.

Declaration

Author Contribution: Conceive-F.N.S.K.; Design- F.N.S.K.; Supervision- F.N.S.K.; Experimental Performance, Data Collection and/or Processing- F.N.S.K.; Analysis and/or Interpretation- F.N.S.K.; Literature Review- F.N.S.K.; Writer-F.N.S.K.; Critical Reviews– F.N.S.K.

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