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Modeling of Optimum Control of the Distance Learning Process

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ABSTRACT

In this study, based on the use of the principle of modal control, the solution to the problem of modeling the optimal control of the process of distance learning is considered. It is assumed that the process of distance learning is described by a finite-difference equation that contains the main parameters of the learning process (the degree of knowledge, the coefficients of forgetting, and residual knowledge). Based on the results obtained, a scheme for modeling the optimal system of the distance learning process in MATLAB/Simulink is proposed.

1. Introduction

Solving complex management problems in the educational system is currently almost impossible without preliminary modeling of learning processes. Control problems as well as methods of mathematical modeling of learning processes have been studied in numerous works [1-4]. In these works, some aspects of the optimal management of educational processes in the university are considered. In particular, the development of an optimal curriculum begins with the creation of a model for managing the processes of interest. Generally, the parameters of the learning process are the degree of knowledge, the coefficients of forgetting, and residual knowledge. In those problems, simulation modeling methods are applied using the principle of a system approach of control theory [5-7]. Therefore, the development of mathematical models for solving problems of analysis and synthesis of the learning process is crucial.

The organization of the distance learning process, including a set of organizational and methodological measures and the object of learning (or control), will be called a system. The management (organization) system is optimal when it has the best quality of the distance learning process in terms of its parameters, the degree of knowledge, the coefficients of forgetting, and residual knowledge, under given conditions and restrictions. The quality of such systems can be assessed by some optimality criterion J, considering the current level of knowledge $x(n)$ (state) and effort cost (or management) for training $g(n)$ for some finite time interval $[0, N]$.

Commonly, a finite-difference equation of the process of distance learning is known as a non-linear vector [8,9]. Restrictions on the control and state of the system are set. It is required to find a control vector (control algorithm) that transfers the control object from the initial state to the final state, keeps it in this final state, or changes it by the input signal while providing an extreme value of the optimality criterion [10,11]. The simulation is considered complete if the control algorithm is found as a function of the vector of state variables under known restrictions on the components of the control vector. The task of modeling an optimal system is to develop a system or control algorithm that minimizes some optimality criterion, considering the constraints on the control and state.

2. Materials and Methods

2.1 Statement of the problem

Here, we study linear and discrete dynamical systems. Let us consider the application of the Z-transform for solving the problems of the analysis of such one-dimensional stationary systems.

Let the process of distance learning of higher mathematics in the engineering discipline be described by an ordinary linear difference equation of the first order:

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$$x(n+1)-(1-a)x(n)=g(n), \tag{1}$$

where $x \in [0,1]$ is the degree of knowledge of elements of mathematics, $a \in [0,1]$ is the forgetting coefficient, $g \in [0,1]$ is the control of the learning process, and $g(n)=\mu(n)+b$, where $\mu(n) \in [0,1]$ is the learning intensity, and $b \in [0,1]$ is the coefficient of residual knowledge, $n \in [0,N]$ is the current learning time. After applying transformations to equation (1), we obtain the equation in deviations as

$$x^*(n)=x(n)-q, \tag{2}$$

where q is the desired level of knowledge. Note that equation (2) is valid for any n, including $n+1$. Analyzing this expression, it is easy to see that in the limit $q=1$ and as $n \to \infty, x^* \to 0$.

However, such a requirement on the average can never be achieved. Optimal management (or organization of learning) can only provide the best approximation in some sense to the desired level.

Considering these, the finite-difference equation of the learning process in (2) can bewritten in the following form:

$$x^*(n+1)-(1-a)x^*(n)=g(n)-aq. \tag{3}$$

It should be noted that the right side of equation (3) represents the optimal control of the learning process, i.e., $g^*(n)=g(n)-aq$. In addition, if we consider the equation in (3), the control of the learning process $g(n)$ is determined, respectively, through the intensity of learning $\mu(n)$ and the coefficient of residual knowledge b. We then obtain

$$x^*(n+1)-(1-a)x^*(n)=\mu(n)+b-aq. \tag{4}$$

For further analysis, we require that the control delivers a minimum quadratic error with minimum training intensity. To solve this problem, we will use the method of variational calculus [5-9].

According to the principle of variation, the problem of finding the extreme values of a function in the presence of constraints in the form of equalities is solved by the method of indefinite Lagrange multipliers. In this case, the optimality criterion can be written as follows:

$$J_c=\sum_n^N F_c[x(n),g(n)], \tag{5}$$

Where

$$F_c[x(n),g(n)]=\frac{1}{2}[x^2(n)+g^2(n)]+\lambda(n+1)[x(n+1)-(1-a)x(n)-g(n)].$$

As a result of the expansion of the function, F_c in a Taylor series and limited to the first terms of the expansion, we will have [9]

$$F_c^*(n)=\frac{1}{2}[x^{*2}(n)+g^{*2}(n)]+\lambda^*(n+1)[x^*(n+1)-(1-a)x^*(n)-g^*(n)], \tag{6}$$

and

$$F_c^*(n-1)=\frac{1}{2}[x^{*2}(n-1)+g^{*2}(n-1)]+\lambda^*(n)[x^*(n)-(1-a)x^*(n-1)-g^*(n-1)]. \tag{7}$$

Next, we define the discrete Euler-Lagrange equation for which the case under consideration has the form

$$\frac{\partial F_c^*(n)}{\partial x^*(n)}+\frac{\partial F_c^*(n-1)}{\partial x^*(n)}=\frac{1}{2}2x^*(n)-(1-a)\lambda^*(n+1)+\lambda^*(n)=0,$$

or

$$x^*(n)-(1-a)\lambda^*(n+1)+\lambda^*(n)=0. \tag{8}$$

In the next step, we obtain the process equation under optimum conditions. To do this, by differentiating expression (4) with respect to the Lagrange multiplier, we obtain

$$\frac{\partial F_c^*(n)}{\partial \lambda^*(n+1)}=x^*(n+1)-(1-a)x^*(n)-g^*(n)=0 \tag{9}$$

We also define the optimal control as follows:

$$\frac{\partial F_c^*(n)}{\partial g^*(n)}=g^*(n)-\lambda^*(n+1)=0,$$

where

$$g^*(n)=\lambda^*(n+1). \tag{10}$$

Equation (8) means that to determine the optimal control, it is necessary to know the law of change in the Lagrange multiplier, $\lambda^*(n+1)$. Based on the joint solution of two first-order difference equations (equations (6) and (7)) and considering the equation in (8), we can determine the Lagrange multiplier, $\lambda^*(n+1)$. Since $x(0)$ and $x(N)$ are given, these values determine the boundary conditions for solving equations (6) and (7). The system of two difference equations has the form:

$$\begin{cases} (1-a)\lambda^*(n+1)-\lambda^*(n)-x^*(n)=0, \\ x^*(n+1)-\lambda^*(n+1)-(1-a)x^*(n)=0, \end{cases} \tag{11}$$

under boundary conditions $x^*(0)$ and $x^*(N)$.

2.2. Solution of the equations of the optimal system by the z-transformation method

We solve equations (11) by the z-transform method. Let's introduce the new notation: $x^*=x$, $\lambda^*=y$. Then, equations (11) can be written as follows:

$$\begin{cases} (1-a)y(n+1)-y(n)-x(n)=0, \\ x(n+1)-y(n+1)-(1-a)x(n)=0. \end{cases} \tag{12}$$

We apply z-transform to the equations in (12) using the time domain shift theorem:

$$\begin{cases} (1-a)[zY(z)-y(0)]-Y(z)= X(z), \\ zX(z)-x(0)-(1-a)X(z)=[zY(z)-y(0)]. \end{cases} \quad (13)$$

Given that $x(0)=1$, we express (13) in its matrix form as:

$$\begin{bmatrix} 1 & (1+az-z) \\ (z+a-1) & -z \end{bmatrix}\begin{bmatrix} X(z) \\ Y(z) \end{bmatrix}=\begin{bmatrix} (a-1)y_0 \\ 1-y_0 \end{bmatrix}. \quad (14)$$

Solving the matrix equation for X and Y, respectively, we will have

$$X(z)=\frac{(1-a)z+y_0-1}{(1-a)z^2-(a^2-2a+3)z+1-a}, \quad (15a)$$

and

$$Y(z)=\frac{(a-1)y_0z+(a^2-2a+2)y_0-1}{(1-a)z^2-(a^2-2a+3)z+1-a} \quad (15b)$$

Further, to simplify the analysis, let's assume that the forgetting coefficient a=0.4. Then, based on equations (15a) and (15b), we will have:

$$X(z)=\frac{0.6\,z+y_0-1}{0.6\,z^2-2.36z+0.6}, \quad (16a)$$

and

$$Y(z)=\frac{y_0z-2.67y_0+1.67}{z^2-3.933z+1}. \quad (16b)$$

To obtain the originals (inverse transformations), we first decompose the equation (16a) and (16b) into the sum of the simplest terms as

$$X(z)=\frac{0.6\,z}{0.6\,z^2-2.36z+0.6}+\frac{y_0-1}{0.6\,z^2-2.36z+0.6}, \quad (17a)$$

and

$$Y(z)=\frac{y_0z}{z^2-3.933z+1}-\frac{2.67y_0-1.67}{z^2-3.933z+1}. \quad (17b)$$

First, the denominators of all expansion terms must be linear in the form of $+p_i$, where p_i is a real non-positive root of the denominator of the equations (17a) or (17b). Thus, having performed the inverse z-transformation for the equations (17a) and (17b), we obtain the original function for them in the following form, respectively:

$$x^*(n)=1.081*3.66^n-0.0807*0.2732^n+(y_0-1)*0.4921*(3.66^n-0.2732^n), \quad (18a)$$

$$y^*(n)=y_0*(1.0807*3.66^n-0.0807*0.2732^n)-(2.67y_0-1)*0.2953*(3.66^n-0.2732^n). \quad (18b)$$

Based on the equations above, to solve the problem, we need to determine the initial condition and the law of optimal control. Let, for concreteness, the final error $x(N)=x(10)=0$. Under these assumptions, using equation (18a), we can find the initial value of y_0. Thus, we will have

$$x^*_{|N=10}(n)=1.081*3.66^{10}-0.0807*0.2732^{10}+(y_0-1)*0.4921*3.66^{10}=0. \quad (19)$$

Further, from the equation in (19) for the initial value y_0, we obtain

$y_0=\lambda^*(0)=-1.1967$

Substituting this initial value into (18a), we find the optimal trajectory as

$$X^*(n)=0.2732^n \quad (20)$$

As shown above, the optimal control is then described by the relation

$g^*(n)=\lambda^*(n+1),$

where, $\lambda^*(n+1)$ satisfies equation (18b), which is valid for any n, including $n+1$. Therefore, substituting it into (18b), we obtain

$y_0=\lambda^*(0)=-1.1967,$

and after performing simple arithmetic transformations, we get

$y^*(n)=\lambda^*(n)=-1.1967*0.2732^n.$

Further in this expression, replacing n with $n+1$, we will have

$\lambda^*(n+1)=-1.1967*0.2732^{n+1}=-0.3269*0.2732^n.$

The resulting expression allows us to have the control law for the distance learning system in a closed form:

$g^*(n)=\lambda^*(n+1)=-0.3269*0.2732^n,$

or

$$g^*(n)=-0.3269*x^*(n)=-C*x^*(n), \quad (21)$$

where $C=0.3269.$

Thus, the task of modeling optimal control has been solved.

3. Results & Discussion

For clarity of the obtained results of the task, Fig. 1 shows the simulation scheme of the optimal system in MATLAB/Simulink. Based on this scheme, the change in the intensity of learning, as well as the change in the degree of knowledge, for the optimal system are shown in Fig. 2 and Fig. 3, respectively.

In MATLAB/Simulink, the change curve of the degree of information determined for the simulation circuit with the same parameters as the optimum system but not optimal is shown in Figure 4.

Comparison of the results shows that by increasing the intensity of training (management), the degree of knowledge of the elements of the discipline increases from 0.16 (in the original non-optimal system) to ~ 0.46 (in the optimal system).

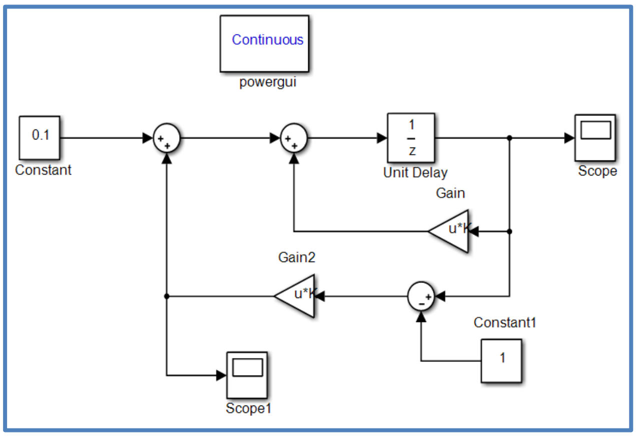

Fig 1. Optimal System Modeling Scheme.

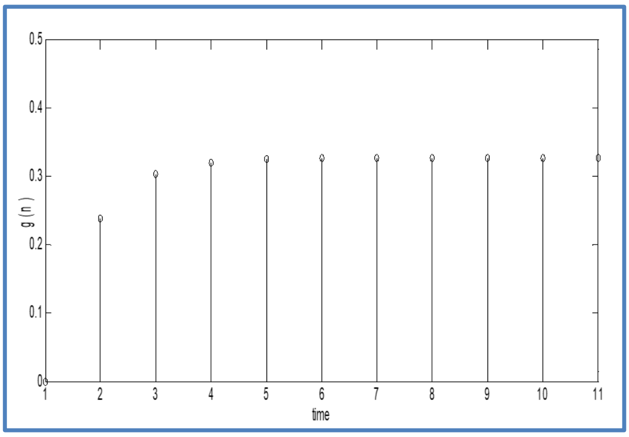

Fig 2. Changing the intensity of the learning (management).

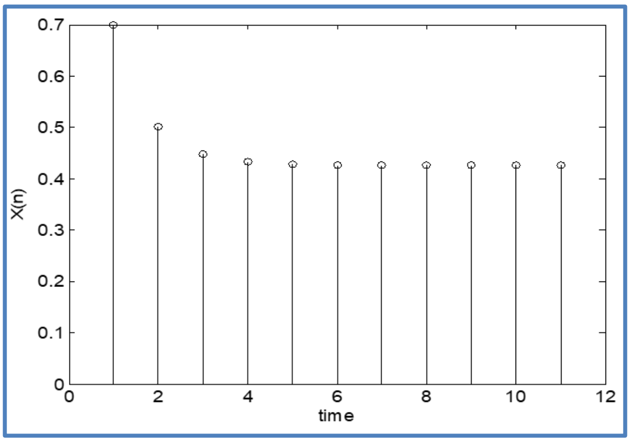

Fig 3. Changing the degree of knowledge (state) in the optimal system.

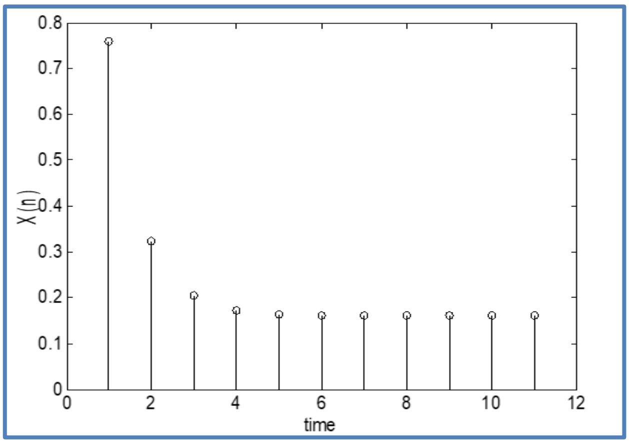

Fig 4. Changing the degree of knowledge in a non-optimal system.

4. Conclusion

In this paper, the problem of modelling the optimal control system for distance learning is formulated. Using the method of indefinite Lagrange multipliers, a system of finite-difference equations for determining the law of optimal control is obtained and its solution is found using the Z-transform.

Under the proposed optimal system, the implementations performed in MATLAB/Simulink show the advantage of the optimal system over the non-optimal one.

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