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Explicit algebraic classification of vacuum, shearfree and non-twisting spacetimes at Large D

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ABSTRACT

Most general, shearfree and twistfree geometry is revisited and its Weyl scalars are obtained with large D limit. It is shown that, the spacetime is Type I(b) or more special in this limit like the classification of any arbitrary dimesion $D > 4$. As an example, classification of vacuum RT spacetime is investigated. As expected, the spacetime becomes algebraically special and it is Type II or more special. Obligatory conditions are determined for other types and sub-types as the dimension of the spacetime $D \to \infty$.

1. Introduction

After Robinson-Trautman (RT) solution in 4-dimensions [1, 2] was obtained, the results which enable to understand blackhole physics, theory of gravitational waves and cosmology were commonly studied. As the spacetime geometrically defines a shear-free, twist-free and expanding congruence of null geodesics, it involves many well-known solutions i.e. Schwarzchild, Reissner–Nordstrom, Schwarzchild–de Sitter, Vaidya, C-metric. So, the RT spacetime in 4 dimensions has include various algebraically special Petrov-Penrose types which are analyzed in [3, 4].

RT solutions in empty space generalized to higher dimensions in [5], but surprisingly higher dimensional results does not include as many as solutions like $D = 4$. Additionally, aligned electromagnetic fields within Einstein-Maxwell theory [6] and general p-form Maxwell fields [7] were associated with higher dimensional RT spacetime to analyze richness of it. After a classification scheme for the Weyl tensor of higher dimensional spaces with Lorentzian signature was put forward [8, 9] (developments and applications of the classification of the Weyl tensor in higher dimensional Lorentzian geometries is reviewed in [10]), classification of higher dimensional RT spacetime was explicitly analyzed [11].

On the other hand, Emparan and et al improved a new perspective to higher dimensional solutions with large D expansion method [12–22]. More concretely, the limit $D \to \infty$ results in a convenient simplification of the equations and possibly also a novel reformulation of the dynamics [23]. Although algebraic classification of RT is studied [24] as the dimension of the spacetime $D \to \infty$, algebraically special Petrov types and subtypes of vacuum RT spacetime are discussed for the first time.

The paper is organized as; Section 2 we revisit higher dimensional shear-free, non-twisting and expanding metric and its Weyl scalars. By defining boost weight and vanishing Weyl scalars the algebraic classification of the general metric is summarized. Main purpose of the paper is studied in Section 3. Algebraic classification of the vacuum RT spacetime for primary and secondary WANDs are investigated with obligatory conditions.

2. Algebraic structure of the Weyl tensor at large D

D dimensional most general, shear-free, twist-free metric can be written in the form [5];

$$ds^2 = g_{pq}(u,r,x)\,dx^p dx^q + 2g_{up}(u,r,x)\,du dx^p - 2du dr + g_{uu}(u,r,x)\,du^2 \quad (1)$$

where latin indices $p, q, ...$ count to 2 to $(D-1)$ and x is shorthand of these $D-1$ spatial coordinates on the traverse space. Non-twisting structure of the spacetime causes a null foliation by null hypersurfaces u =const. which ensures to define the coordinate u. Equivalently, a non-twisting null vector field $\mathbf{k}$ that is everywhere tangent (and normal) to u =const. can be defined. So, the affine parameter r along a null geodesic congruence generated by $\mathbf{k}$ is determined as the second coordinate which gives $\mathbf{k} = \partial_r$. The relations between covariant and contravariant metric components of the metric 1 become; $g^{ur} = -1, g^{rp} = g^{pq} g_{uq}, g^{rr} = -g_{uu} + g^{pq} g_{up} g_{uq}$. In addition, the traverse space metric can be introduced as $g_{pq} = R^2(u,r,x) h_{pq}(u,x)$ where $R = exp\left(\int \Theta(u,r,x)dr\right)$ and Θ is corresponding to the expansion. Although $\Theta = 0$ (non-expanding case) corresponding Kundt spacetime, we will analyze expanding case that is RT spacetime.

The most natural null frames for the metric (1) can be written in the form;

$$\mathbf{k} = \partial_r, \quad \boldsymbol{\ell} = \frac{1}{2} g_{uu}\partial_r + \partial_u, \quad \mathbf{m_i} = m_i^p\left(g_{up}\partial_r + \partial_p\right), \quad (2)$$

where they satisfy the normalization conditions; $\mathbf{k}.\boldsymbol{\ell} = -1$, $\mathbf{m_i}.\mathbf{m_j} = \delta_{ij}$. By rescaling of these null frames $\mathbf{k} \to \lambda \mathbf{k}$, $\boldsymbol{\ell} \to \lambda^{-1}\boldsymbol{\ell}$ and $\mathbf{m}_i \to \mathbf{m}_i$, boosts are obtained. One summarizes the boost weight of the null basis $+1, -1, 0$, respectively [25].

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Weyl scalar components of the metric (1) for the null frame can be obtained with large D expansion as;

$$\Psi_{0^{ij}} = C_{abcd}k^a m_i^b k^c m_j^d = m_i^p m_j^q C_{rprq} = 0, \quad (3)$$

$$\Psi_{1T^i} = C_{abcd}k^a \ell^b k^c m_i^d = m_i^p C_{rurp}$$
$$= m_i^p \left[\left(-\frac{1}{2} g_{up,r} + \Theta g_{up} \right)_{,r} + \Theta_{,p} \right], \quad (4)$$

$$\Psi_{1^{ijk}} = C_{abcd}k^a m_i^b m_j^c m_k^d = m_i^p m_j^q m_k^m C_{prmq} = 0, \quad (5)$$

$$\Psi_{2S} = C_{abcd}k^a \ell^b \ell^c k^d = C_{ruur}$$
$$= \left(\frac{1}{2} g_{uu,r} - \Theta g_{uu} \right)_{,r} - \frac{1}{4} g^{pq} g_{up,r} g_{uq,r} - 2\Theta_{,u}$$
$$+ \Theta g^{rp} g_{up,r} - \Theta^2 g^{rp} g_{up}, \quad (6)$$

$$\Psi_{2T^{ij}} = C_{abcd}k^a m_i^b \ell^c m_j^d = m_i^p m_j^q \big(C_{rpuq} + g_{up} C_{rqur}$$
$$+ \frac{1}{2} g_{uu} C_{rprq} \big)$$
$$= m_i^p m_j^q \bigg(\frac{1}{2} g_{up} g_{uq,rr} + \frac{1}{4} g_{up,r} g_{uq,r} + \frac{1}{2} g_{pn} g^{ms} g_{us,r} \, {}^s\Gamma^n_{mq}$$
$$+ \frac{1}{2} g_{pn} \left(g^{nm} g_{um,r} \right)_{,q}$$
$$+ g_{up} g_{uq} \left(\Theta^2 - \Theta_{,r} \right) - 2 g_{up} \Theta_{,q} - \Theta \left(2E_{qp} - g_{uq} g_{up,r} \right) \bigg) \quad (7)$$

$$\Psi_{2^{ijkl}} = C_{abcd} m_i^a m_j^b m_k^c m_l^d = m_i^p m_j^q m_k^n m_l^m \big(C_{pqmn}$$
$$+ g_{up} C_{rqmn} + g_{uq} C_{prmn} + g_{um} C_{pqrn}$$
$$+ g_{un} C_{pqmr} \big) = m_i^p m_j^q m_k^n m_l^m C_{pqnm}, \quad (8)$$

$$\Psi_{2^{ij}} = C_{abcd} k^a \ell^b m_i^c m_j^d = m_i^p m_j^q \big(C_{rupq} + g_{uq} C_{rurp}$$
$$+ g_{up} C_{ruqr} \big)$$
$$= m_i^p m_j^q \bigg(g_{u[p,q]r} - 4 g_{u[p} \Theta_{,q]}$$
$$+ g_{u[p} g_{q]u,rr} + \Theta \left(2 g_{u[q} g_{p]u,r} + E_{qm} - E_{pn} \right) \bigg), \quad (9)$$

$$\Psi_{3T^i} = C_{abcd} \ell^a k^b \ell^c m_i^d = m_i^p \big(\frac{1}{2} g_{uu} C_{urrp}$$
$$+ g_{up} C_{urur} + C_{urup} \big) = m_i^p \bigg(\frac{1}{4} g_{uu} g_{up,rr}$$
$$- g_{u[u,p]r} + g_{up} \Theta_{,u} + \frac{1}{2} g_{uu} \Theta_{,p} - \frac{1}{2} g_{uu} g_{up} \Theta_{,r}$$
$$+ \frac{1}{2} g^{mn} g_{um,r} E_{np} - g_{up} \big(\frac{1}{2} g_{uu,r} - \Theta g_{uu} \big)_{,r}$$
$$- \frac{\Theta}{2} \left(g^{rr} g_{up,r} + g_{uu,p} + 2 g^{rs} E_{sp} \right) \bigg), \quad (10)$$

$$\Psi_{3^{ijk}} = C_{abcd} \ell^a m_i^b m_j^c m_k^d = m_i^p m_j^q m_k^m \bigg(\frac{1}{2} g_{uu} \big(C_{rpqm}$$
$$+ g_{uq} C_{rprm} + g_{um} C_{rpqr} \big) + g_{up} \big(C_{urqm}$$
$$+ g_{uq} C_{urrm} + g_{um} C_{urqr} \big) + g_{uq} C_{uprm} + g_{um} C_{upqr}$$
$$+ C_{upqm} \bigg) = m_i^p m_j^q m_k^m \bigg(- 2 g_{up} g_{u[q,m]r}$$
$$- \Theta g_{up} \left(E_{mn} - E_{qs} \right) + g_{up} g_{u[q} g_{m]u,rr} + g_{up} g_{u[q} g_{m]u,r}$$
$$+ g^{\ell s} g_{us,r} \, {}^s\Gamma^n_{\ell p} g_{u[q} g_{m]n} + \frac{1}{2} g_{up,r} g_{u[q} g_{m]u,r}$$
$$+ g_{u[q} g_{m]n} \left(g^{n\ell} g_{u\ell,r} \right)_{,p} - 4\Theta E_{p[m} g_{q]u}$$
$$- g_{pn} \left(g^{rn} g_{u[m,r)} \right)_{,q]} - E_{p[m} g_{q]u,r} - 2 g_{pn} \left(g^{ns} E_{s[q)} \right)_{,m]}$$
$$- g_{pn} g^{rs} \, {}^s\Gamma^n_{s[q} g_{m]u,r}$$
$$- 2 g_{pn} g^{sk} E_{k[q} \, {}^s\Gamma^n_{m]s} - 2\Theta^2 g^{rr} g_{p[q} g_{m]u} \bigg), \quad (11)$$

$$\Psi_{4^{ij}} = C_{abcd} \ell^a m_i^b \ell^c m_j^d = m_i^p m_j^q \bigg(\frac{g_{uu}}{2} \big(C_{rpuq} + C_{uprq}$$
$$+ \frac{g_{uu}}{2} C_{rprq} + g_{uq} C_{rpur} + g_{up} C_{urrq} \big)$$
$$+ g_{up} \left(C_{uruq} + g_{uq} C_{urur} \right) + g_{uq} C_{upur} + C_{upuq} \bigg)$$
$$= m_i^p m_j^q \bigg(- g_{uq} g_{u[u,p]r}$$
$$+ \frac{g_{uu}}{2} \bigg(g^{ms} g_{us,r} \, {}^s\Gamma^n_{m(p} g_{q)n} + \left(g^{nm} g_{um,r} \right)_{,(p} g_{q)n}$$
$$- 4\Theta E_{(pq)} \bigg) + g^{mn} g_{um,r} E_{n(p} g_{q)u} - \frac{g^{rm}}{2} g_{um,r} g_{u(p} g_{q)u,r}$$
$$+ g_{up} g_{uq} \left(\frac{1}{2} g^{pq} g_{up,r} g_{uq,r} + \Theta g_{uu,r} \right)$$
$$- g_{pn} \big(- \frac{{}^s\Gamma^n_{sq}}{2} \left(g^{rs} g_{uu,r} + 2 g^{sm} E_{um} \right) + \left(g^{rn} g_{u[u,r)} \right)_{,q]}$$
$$- 2 \left(g^{nm} E_{m[q]} \right)_{,u]} \big) + \Theta \big(g_{uu} \left(g_{u(p} g_{q)u,r} + g^{rr} g_{pq,r} \right)$$
$$- E_{u(p} g_{q)u} - g^{rr} g_{u(p} g_{q)u,r} - 2 g_{uu,(p} g_{q)u} \big)$$
$$- \Theta^2 g_{uu} \left(g_{up} g_{uq} + g^{rr} g_{pq} \right) \bigg), \quad (12)$$

where $E_{pq} = g_{u[p,q]} + \frac{1}{2} g_{pq,u}$, and $E_{up} = g_{u[p,u]} + \frac{1}{2} g_{up,u}$. Christoffel symbols, Riemann and Ricci tensors, Ricci scalar and Weyl tensor of the metric (1) are shown in Appendix of our previous work [24] as the dimension of the spacetime $D \to \infty$. Irreducible components of the Weyl scalars stay same at large D limit and the symmetric part of the $\Psi_{2T^{ij}}$ becomes;

$$\Psi_{2T^{(ij)}} = m_i^p m_j^q \big(\frac{1}{2} g_{pn} g^{ms} g_{us,r} \, {}^s\Gamma^n_{mq} + \frac{1}{4} g_{up,r} g_{uq,r}$$
$$+ \frac{1}{2} g_{pn} \left(g^{nm} g_{um,r} \right)_{,q} + g_{up} g_{uq} \left(\Theta^2 - \Theta_{,r} \right)$$
$$- 4 g_{u(p} \Theta_{,q)} + \Theta g_{u(q} g_{p)u,r} + g_{u(p} g_{q)u,rr} \big). \quad (13)$$

Automatically, since the $\Psi_{0^{ij}}$ and $\Psi_{1^{ijk}}$ vanish RT spacetime is classified Type I(b). But it does not algebraically special while the all boost weight of +1 does not vanish. Spacetime will be algebraically special and Type II (equivalently Type I(a)) when the $\Psi_{1T^i} = 0$. In addition, vanishing components of the Weyl scalar and corresponding types and subtypes of the RT spacetime for the primary and secondary WANDs $\mathbf{k}, \boldsymbol{\ell}$ are summarized at the Table 1.

3. Analysis of vacuum Robinson-Trautman spacetime

Most general vacuum, shear-free, non-twisting metric as the dimension of the spacetime $D \to \infty$ can be written;

$$ds^2 = r^2 h_{pq}(u,x) dx^p dx^q + r^2 d^p(u,x) d_p(u,x) du^2$$
$$+ 2r^2 d_p(u,x) du dx^p - 2 du dr \quad (14)$$

where the expansion $\Theta = \frac{1}{r}$ and the components of the metric $g_{pq} = r^2 h_{pq}(u,x)$, $g^{rr} = 0$, $g_{up} = r^2 d_p$, $g^{rp} = d^p$. General form of the metric component g^{rr} is given equation 100 in [11]. Interestingly, it vanishes with large D limitation. As a result, our metric does not include the cosmological constant and pure radiation terms. Since the component of Weyl scalar Ψ_{1T^i} becomes zero, the spacetime is obtained algebraically special and **Type II and more special**. Non-zero components of the Weyl scalars can be written;

$$\Psi_{2S} = d^p d_p, \quad (15)$$

$$\Psi_{2T^{(ij)}} = m_i^p m_j^q \Big[h_{pn} r \left[h^{ms} d_s \, {}^s\Gamma^n_{mq} + \left(h^{nm} d_m \right)_{,q} \right]$$

Types	Vanishing Weyl Scalar
I	$\Psi_{0^{ij}}$
I(a)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}$
I(b)	$\Psi_{0^{ij}}, \Psi_{1^{ijk}}$
II	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}$
II(a)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}$
II(b)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2T^{(ij)}}$
II(c)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2^{ijk\ell}}$
II(d)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2^{ij}}$
III	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}}$
III(a)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}}, \Psi_{3T^{i}}$
III(b)	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}}, \Psi_{3^{ijk}}$
N	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}}, \Psi_{3T^{i}}, \Psi_{3^{ijk}}$
O	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}}, \Psi_{3T^{i}}, \Psi_{3^{ijk}}, \Psi_{4^{ij}}$
I_i	$\Psi_{0^{ij}}, \Psi_{4^{ij}}$
II_i	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{4^{ij}}$
III_i	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{2S}, \Psi_{2T^{(ij)}}, \Psi_{2^{ijk\ell}}, \Psi_{2^{ij}},, \Psi_{4^{ij}}$
D	$\Psi_{0^{ij}}, \Psi_{1T^{i}}, \Psi_{1^{ijk}}, \Psi_{3T^{i}}, \Psi_{3^{ijk}}, \Psi_{4^{ij}}$

Table 1. Algebraic classification of the RT geometry for the primary and secondary WANDs $\mathbf{k}, \boldsymbol{\ell}$ [11].

$$+7r^2 d_p d_q \Bigg], \tag{16}$$

$$\Psi_{2^{ijk\ell}} = m_i^p m_j^q m_k^n m_l^m \tilde{C}_{pqnm}, \tag{17}$$

$$\Psi_{2^{ij}} = m_i^p m_j^q \left[2r d_{[p,q]} + \frac{1}{r}\left(E_{qm} - E_{pn}\right) \right], \tag{18}$$

$$\Psi_{3T^i} = m_i^p \Bigg[r d_t d^t d_{p,u} - \frac{3r}{2}\left(d_t d^t\right)_{,p} - \frac{d_s}{r} E_{sp} + \frac{1}{r} d_m h^{mn} E_{np} + r^2 d_p d_t d^t \Bigg], \tag{19}$$

$$\begin{aligned}\Psi_{3^{ijk}} = m_i^p m_j^q m_k^n \Bigg[& -4r^3 d_p d_{[q,m]} - r d_p \left(E_{mn} - E_{qs}\right) \\ & -2r^3 d_{[q} h_{m]n} \left(h^{\ell s} d_s\, {}^s\Gamma^n_{\ell p} - d^n_{,p} \right) \\ & -6r E_{p[m} d_{q]} - 2r^3 h_{pn} \left[\left(d^n d_{[m}\right)_{,q]} + d^s\, {}^s\Gamma^n_{s[q} d_{m]} \right] \\ & -2h_{pn} \left[\left(h^{ns} E_{s[q}\right)_{,m]} + h^{sk} E_{k[q}\, {}^s\Gamma^n_{m]s} \right] \Bigg],\end{aligned} \tag{20}$$

$$\begin{aligned}\Psi_{4^{ij}} = m_i^p m_j^q \Bigg[& - r^3 \left(2d_q \left(d_t d^t\right)_{,p} - d_q d_{p,u} \right) \\ & +4 d_t d^t \left(r^3 h^{ms} d_s\, {}^s\Gamma^n_{m(p} h_{q)n} + r^3 \left(h^{nm} d_m\right)_{,(p} h_{q)n} \right. \\ & \left. - E_{(pq)} \right) + 2h^{mn} d_m E_{n(p} d_{q)} - 8r^4 d^p d_p d_q d_m \\ & + r^4 d_p d_q \left(2h^{pq} d_p d_q + 5 d_t d^t\right) \\ & - r^2 h_{pn} \left[- {}^s\Gamma^n_{sq} \left(r d^s d_t d^t + \frac{h^{sm}}{r^2} E_{um} \right) \right. \\ & \left. + r \left[\left(d^n d_t d^t\right)_{,q} - \left(d^n d_q\right)_{,u} \right] - \frac{2}{r^2}\left(h^{nm} E_{m[q}\right)_{,u]} \right] \\ & - r E_{u(p} d_{q)} - r^3 d_p \left(d^t d_t\right)_{,q} \Bigg],\end{aligned} \tag{21}$$

where $E_{pq} = r^2\left(d_{[p,q]} + \frac{1}{2}h_{pq,u}\right)$, $E_{up} = r^2 d_{p,u} - r^2\left(d^t d_t\right)_{,p}$ and ${}^s\Gamma^n_{pq} = \frac{1}{2}h^{nt}\left(h_{pt,q} + h_{qt,p} - h_{pq,t}\right)$ and $\tilde{C}_{pqnm}$ corresponding Weyl tensor of traverse space. Weyl tensors are became more simpler than the results of any arbitrary dimensions $D > 4$ [11, 26].

Then, we can easily determine the obligatory conditions for types and sub-types of the vacuum RT solution by using Table 1.

- Vacuum RT solution will be sub-type Type II(a) if the Weyl scalar $\Psi_{2S} = 0$, which yields;

$$d_p d^p = 0. \tag{22}$$

- Vacuum RT solution will be sub-type Type II(b) if the Weyl scalar $\Psi_{2T^{(ij)}} = 0$ which gives;

$$d^n_{,q} = -d^m\, {}^s\Gamma^n_{mq} \quad \text{and} \quad d_p d_q = 0. \tag{23}$$

- Vacuum RT solution will be sub-type Type II(c) if the Weyl scalar $\Psi_{2^{ijk\ell}} = 0$, which yields;

$$\tilde{C}_{pqnm} = 0. \tag{24}$$

- Vacuum RT solution will be sub-type Type II(d) if the Weyl scalar $\Psi_{2^{ij}} = 0$, which yields;

$$d_{[p,q]} = \frac{1}{2r^2}\left(E_{pn} - E_{qm}\right). \tag{25}$$

- Vacuum RT solution will be Type III (equivalently sub-type Type II(abcd)) if the above Weyl scalars vanish and equations (22)-(25) are satisfied simultaneously. Additionally, the spacetime becomes Type III(a) because the Weyl scalar Ψ_{3T^i} is obtained zero with these conditions.
- Vacuum RT solution will be sub-type Type III(b) and Type N if the Weyl scalar $\Psi_{3^{ijk}} = 0$, which yields;

$$\begin{aligned} & d_{[q} h_{m]n} = 0, \qquad E_{p[m} d_{q]} = 0, \\ & \left(h^{ns} E_{s[q}\right)_{,m]} = -h^{sk} E_{k[q}\, {}^s\Gamma^n_{m]s}. \end{aligned} \tag{26}$$

- Vacuum RT solution will be Type O if all above conditions are satisfied with the Weyl scalar $\Psi_{4^{ij}} = 0$, which yields;

$$\begin{aligned} & d_q d_{p,u} = 0, \qquad E_{u(p} d_{q)} = 0, \\ & \left(h^{nm} E_{m[q}\right)_{,u]} = -h^{sm} E_{um}\, {}^s\Gamma^n_{sq}. \end{aligned} \tag{27}$$

  Final condition requires the coefficient of d_p independent of the parameter u ($d_p(u,x) \to d_p(x)$).

The algebraic classification of vacuum RT geometry and obligatory conditions are summarized at Table 2 for primary WAND $\mathbf{k}$. Various combinations of sub-types can be obtained by using the neccassary conditions such as Type II(ad) occur if equations (22) and (25) are satisfied simultaneously.

The secondary alignment Types I_i and II_i arise when the equation (21) vanishes which yields $Ar^4 + Br^3 - Cr + D = 0$ where

$$A = -8d^p d_p d_q d_m + d_p d_q (2h^{pq} d_p d_q + 5 d^t d_t) \tag{28}$$

$$\begin{aligned} B = & -(2d_q (d^t d_t)_{,p} - d_p d_{p,u}) + 4 d_t d^t h^{ms} d_s\, {}^s\Gamma^n_{m(p} h_{q)n} \\ & + (h^{mn} d_m)_{,(p} h_{q)n} + h_{pn} d^s d_t d^t\, {}^s\Gamma^n_{sq} + (d^n d_t d^t)_{,q} \\ & - (d^n d_q)_{,u} - d_p (d^t d_t)_{,q} \end{aligned} \tag{29}$$

$$C = E_{u(p} d_{q)} \tag{30}$$

$$D = h_{pn} h^{sm} E_{um}\, {}^s\Gamma^n_{sq} + 2h_{pn}(h^{nm} E_{m[q})_{,u]}. \tag{31}$$

In addition, if the equations (22-25) are satisfied with $d_p(u,x) \to d_p(x)$, the spacetime becomes Type III_i for the secondary WAND $\boldsymbol{\ell}$.

Types	Obligatory Conditions
I	always
I(a)	always
I(b)	always
II	always
II(a)	$d_p d^p = 0$
II(b)	$d^n_{,q} = -d^m\, {}^s\Gamma^n_{mq}$ and $d_p d_q = 0$
II(c)	$\tilde{C}_{pqnm} = 0$
II(d)	$d_{[p,q]} = \frac{1}{2r^2}\left(E_{pn} - E_{qm}\right)$
III	all above conditions
III(a)	all above conditions
III(b) and N	all above conditions , $d_{[q}h_{m]n} = 0, \quad E_{p[m}d_{q]} = 0,$ $\left(h^{ns}E_{s[q}\right)_{,m]} = -h^{sk}E_{k[q}\, {}^s\Gamma^n_{m]s}$
O	all above conditions , $d_{p,u} = 0 \leftrightarrow d_p(u,x) \rightarrow d_p(x)$
I_i and II_i	$A = B = C = D = 0$
III_i	$d_p d^p = 0$, $d^n_{,q} = -d^m\, {}^s\Gamma^n_{mq}$ and $d_p d_q = 0$ $\tilde{C}_{pqnm} = 0$, $d_{[p,q]} = \frac{1}{2r^2}\left(E_{pn} - E_{qm}\right)$ $d_{p,u} = 0 \leftrightarrow d_p(u,x) \rightarrow d_p(x)$

Table 2. Algebraic classification of the vacuum RT spacetime with obligatory conditions for the primary and secondary WAND $\mathbf{k}$ and $\boldsymbol{\ell}$.

4. Conclusion

We revisited the most general Robinson Trautman metric in higher dimensions. By defining the most natural null frames, components of Weyl scalar were given at large D. Because $\Psi_{0^{ij}}$ and $\Psi_{1^{ijk}}$ vanished, RT spacetime is Type I(b) or more special. Additionally, vanishing components of Weyl scalar are shown at Table 1 to identify types and sub-types of the RT geometry. In addition, vacuum RT spacetime becomes Type II or more special as the dimension of the spacetime $D \rightarrow \infty$, like results of any arbitrary dimension $D > 4$. It should not be forgotten that, cosmological constant and pure radiation terms vanish with this limitation. Petrov types and subtypes of vacuum RT geometry were accurately investigated, and obligatory conditions were determined for primary and secondary WANDs $\mathbf{k}$ and $\boldsymbol{\ell}$ in Table 2. In future studies, by solving field equations, the property of the limitation method can be fully obtained.

Decleration

Author Contribution: Conceive-P.K.; Design-P.K.; Supervision-P.K.; Computational Performance, Data Collection and/or Processing-P.K.; Analysis and/or Interpretation Literature Review-P.K.; Writer-P.K.; Critical Reviews–P.K.;

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