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The Frequency Response of the Butterworth Filter Approximation

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ABSTRACT

This study creates the design of a 10th order Butterworth bandpass filter based on an operational amplifier. It also provides the analysis of the results obtained in this context. Higher order filters are constructed using a combination of second and third order filters. Four parameters need to be specified in order to design a Butterworth bandpass filter, namely Ap (passband attenuation in dB), As (stopband attenuation in dB), fp (frequency at which Ap occurs), and fs (frequency at which As occurs). The design procedure involves two steps: one is to find the required filter order, and the other is to find the scaling factor which is to be applied to the formatted parameter values. The Butterworth bandpass filter is a combination of a low-pass filter and a high-pass filter. For a Butterworth low-pass filter, the resistance is approximately of 100 kΩ and the capacitor value is found by inverse scaling with the frequency and the chosen resistance value. For a Butterworth high pass filter, the used capacitor value is approximately of 0.05 µF and the resistor value is found by inverse scaling with the frequency and the chosen resistance value.

1. Introduction

One of the most important technical problems in telecommunication systems is related to signal conversion. One of the main issues here is to ensure the selection or suppression of certain signal frequencies. Devices that perform this conversion are called electrical filters [1-5]. Electrical filters are frequency-selective devices that pass or delay signals in certain frequency ranges.

Electrical filters can be classified by a number of properties, such as the type of amplitude-frequency characteristic (LPF, HPF, PF), as well as by the polynomials used to approximate the transfer function (Butterworth, Bessel, Chebyshev) and by element base (passive and active). They are used to create frequency channels in switching systems, and for separation and conversion of electrical signals [6-10].

The first simple electrical filters, consisting of a coil and a capacitor, served to separate telegraph and telephone signals transmitted over wires. Since then, the theory and technology of electrical filters have been continuously improved and remain relevant as one of the most important problems of modern, rapidly developing technology [12-16].

2. Material and Method

2.1. Metod of Designing Filtration Devices Based on Operating Parameters

A bandpass filter is a filter that passes frequencies which are in a certain frequency band between the boundary left and right frequencies. The range of frequencies of transmitted oscillations, for which the absolute value of the transmission coefficient is equal to a certain value with a given accuracy, is called the passband (PB). The range of frequencies of delayed oscillations, for which the absolute value of the transmission coefficient does not exceed a certain specified value, is called the stopband (SB). Between the passband and the stopband lies the transition region (TR) [17-22].

The method of calculating a filter based on operating parameters allows one to obtain a given frequency response of the filter, which is based on the operating attenuation characteristic (OAC)

$$a(\omega) = 20lg\left(\frac{1}{|K(\omega)|}\right) = 10lg\,(1 + \varepsilon^2 \psi\,(\omega)) \qquad (1)$$

where $a(\omega)$ - is the operating attenuation characteristic, dB;
$|K(\omega)|$ - is the filter transmission coefficient modulus, dB;
ε - bandwidth unevenness, % ;
$\psi\,(\omega)$ - filter function (approximation in the passband).
The general form of the operating attenuation characteristic of a bandpass filter is shown in Fig. 1.

As it can be seen from Fig. 1, the characteristic of the working attenuation of the bandpass filter has five regions: the first is the lower stopband, the second is the lower transition region, the third is the passband, the fourth is the upper

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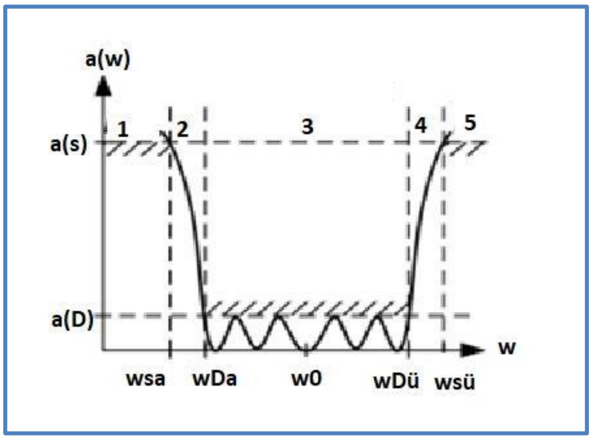

Fig. 1 General view of the operating attenuation characteristic of a bandpass filter.

transition region, and the last one, the fifth, is the upper stopband. Here $\omega_{sL}(\omega_{sH})$ is the lower (upper) cutoff frequency of the stopband; $\omega_{DL}(\omega_{DH})$ is the lower (upper) cutoff frequency of the passband; a_s is the guaranteed transmission in the stopbands; and a_D is the unevenness of the characteristic of the working attenuation in the passband. The characteristic is considered to be specified if all the given parameters are known.

At the initial stage of designing a bandpass filter, we must transform the prototype low-pass filter model into a bandpass filter model. Then comes the stage of normalizing the coefficients. The method of calculating a filter based on operating parameters uses a normalized calculation, in which the real frequency range is normalized relative to the geometric mean center frequency. Next, the operation of denormalizing the coefficients of the resulting PF is carried out. The calculation and construction of the frequency-time characteristics of the normalized and denormalized models are performed afterwards. The geometric mean center frequency of the passband is found by the formula

$$f_0 = \sqrt{f_{ortH*}f_{ortL}} \tag{2}$$

The transfer function of a $K_{LPF}(p)$ bandpass filter can be obtained from the transfer function of a normalized lowpass filter by replacing p with $A \cdot (p + \frac{1}{p})$. In this expression, A - is a transformation constant that characterizes the relative bandwidth of the filter.

$$K_{LPF}(p) = \frac{1}{C(p+a)\prod_{v=1}^{k} \quad (p^2 + 2\alpha_v p + \gamma_v)} \tag{3}$$

where

$$\gamma_v = \alpha^2{}_v + \beta^2{}_v$$

By performing the necessary operations and making some transformations, we obtain the following model for a reduced BPF

$$K_{BPF}(p) = \frac{p^{2k+1}}{C_{BGF}\prod_{i=1}^{2k+1} \quad (p^2 + 2\alpha_{BGF_i} p + \gamma_{BGFi})} \tag{4}$$

or

$$K_{BPF}(p) = \frac{p^{2k+1}}{C_{BGF}\prod_{m=1}^{2} \quad (p + \alpha_{BGF_m})\prod_{i=1}^{2k} \quad (p2 + 2\alpha_{BGF_i} p + \gamma_{BGF_i})} \tag{5}$$

when $a^2 - 4A^2 > 0$ formula (5) is used

$$a_{BPF_1} = \frac{a}{2A} - \frac{\sqrt{a^2 - 4A^2}}{2A} \qquad , a_{BPF_2} = \frac{a}{2A} + \frac{\sqrt{a^2 - 4A^2}}{2A} \tag{6}$$

When $a^2 - 4A^2 < 0$ formula (4) is used

$$\alpha_{BPF_{2k+1}} = \frac{a}{2A} \qquad , \beta_{BPF_{2k+1}} = \frac{\sqrt{4A^2 - a^2}}{2A} \tag{7}$$

The coefficients of the operator transfer function required for this situation are found using the following formulas

$$a_{BPF_i} = \begin{cases} \dfrac{\alpha_v - \sqrt[4]{(\alpha_v^2 - \beta_v^2 - 4A^2)^2 + 4\alpha_v^2\beta_v^2} \quad \cos\left(\dfrac{arctg\left(\dfrac{2\alpha_{v\beta_v}}{\alpha_v^2 - \beta_v^2 - 4A^2}\right)}{2}\right.}{2A} & i = 1,3,\dots,2k-1;\ v \\ = \dfrac{i+1}{2} \quad \dfrac{\alpha_v + \sqrt[4]{(\alpha_v^2 - \beta_v^2 - 4A^2)^2 + 4\alpha_v^2\beta_v^2} \quad \cos\left(arctg\left(\dfrac{2\alpha_{v\beta_v}}{\alpha_v^2 - \beta_v^2 - 4A^2}\right)/2\right.}{2A} & \\ i = 2,4,\dots,2k;\ v = i/2 & \end{cases} \tag{8}$$

$$\beta_{BPF_i} = \begin{cases} \dfrac{\beta_v - \sqrt[4]{(\alpha_v^2 - \beta_v^2 - 4A^2)^2 + 4\alpha_v^2\beta_v^2}\ sin\left(\dfrac{arctg\left(\dfrac{2\alpha_{v\beta_v}}{\alpha_v^2 - \beta_v^2 - 4A^2}\right)}{2}\right.}{2A} & i = 1,3, \dots ,2k - 1;\ v = \dfrac{i+1}{2} \\ \dfrac{\beta_v + \sqrt[4]{(\alpha_v^2 - \beta_v^2 - 4A^2)^2 + 4\alpha_v^2\beta_v^2}\ \ sin\left(arctg\left(\dfrac{2\alpha_{v\beta_v}}{\alpha_v^2 - \beta_v^2 - 4A^2}\right)/2\right.}{2A} & i = 2,4, \dots ,2k;\ v = i/2 \end{cases} \quad (9)$$

The coefficient C_1 of the transfer function of the bandpass filter is calculated as follows

$$C_1 = c * A^5 \quad (10)$$

After having performed various transformations with the transfer function of the LPF prototype, calculated the central frequency, found the transformation constant and new coefficients of the operator transfer function, we obtain the transfer function of the bandpass filter and use it to plot the characteristics in normalized form. Before plotting the graphs of the characteristics in denormalized form, we need to denormalize the necessary coefficients of the operator transfer function.

The denormalization operation corresponds to replacing the variable p in expression (3) with $\frac{p}{f_0}$, where f_0 -is the geometric mean central frequency of the passband. By performing the necessary transformations, we obtain denormalized filter models of the form

$$K_{BPF_D}(p) = \frac{p^{2k+1}}{C_{BPF_D} \prod_{i=1}^{2k+1} \quad (p^2 + 2\alpha_{BPF_{Di}}p + \gamma_{BPF_{Di}}} \quad (11)$$

$$K_{BPF_D}(p) = \frac{p^{2k+1}}{C_{BPF_D} \prod_{m=1}^{2} \quad (p + \alpha_{BPF_D}) \prod_{i=1}^{2k} \quad (p^2 + 2\alpha_{BPF_{Di}}p + \gamma_{BPF_{Di}})} \quad (12)$$

The following notations are used in this study

$\omega_d = 2*\pi * f_0$; $c_d = \frac{c_1}{\omega_d^3}$; $\alpha_d = \alpha * \omega_d$; $\beta_d = \beta * \omega_d$; $\gamma_d = \gamma * \omega_d^2$

After finding a new transfer function, it is possible to construct all the necessary characteristics. Based on the obtained transfer function, we can determine the structure of the designed filter, which is obtained by cascading the second and (or) first order links.

As it is known, any filter obtained by cascading the same type of structural links can be described by a complex transfer coefficient:

$$K(pd) = K_1(p) * K_2(p) * K_3(p) * \dots * K_k(p) \quad (13)$$

Consequently, the designed bandpass filter with the transfer function would consist of five second-order links.

2.2. Derivation of Transfer Functions of Filter Links Based on the Rauch Structure

Let us build a basic diagram of a tenth-order bandpass filter on an operational amplifier using the Rauch structure. In order to derive the transfer function of a bandpass filter using the Rauch structure, we will consider second-order filters that will be connected in cascade (see Fig. 2). The bandpass filter passes signal components with frequencies lying between the left and right cutoff frequencies, and delays the rest. Based on this, the presence of separating capacitors in the branches of the circuit is necessary. In order to determine in which branch they should be, we should first put conductivity in all branches.

Let us find the transfer function of each cascade

$$K_{(p)} = \frac{U_{out}}{U_{in}}$$

For this circuit, based on Kirchhoff's law, we will have

$$\left(U_{in} - \frac{Y_5 * (-U_{out})}{Y_4}\right) * Y_1 = \left(\frac{Y_5 * (-U_{out})}{Y_4} - U_{out}\right) Y_3 + \frac{Y_5 * (-U_{out})}{Y_4} * Y_4 + \frac{Y_5 * (-U_{out})}{Y_4} * Y_2 \quad (14)$$

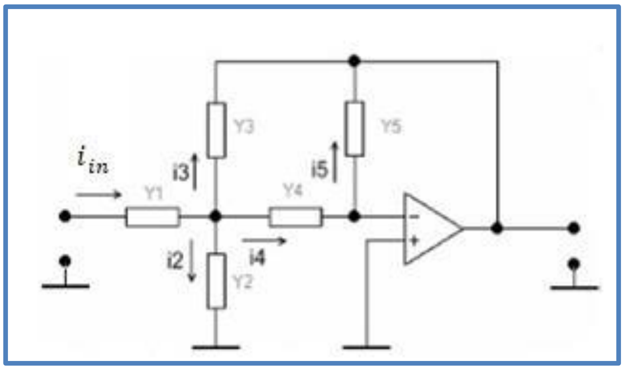

Fig. 2 Functional diagram of the second-order Rauch structure.

The general formula for the transfer characteristic of a bandpass filter is

$$K(\omega) = \frac{j*\omega}{cR*[(j*\omega)^2+2*\alpha_{1d}*j*\omega+\gamma_{1d}]} \tag{15}$$

$$cR = R_2C_3 \cdot \frac{C_1+C_2}{C_1}, \quad 2\alpha = \frac{R_1+R_2}{R_1R_2(C_1+C_2)}, \quad \gamma = \frac{1}{R_1R_2R_3(C_1+C_2)} \tag{18}$$

Taking (16) into account, we can construct a circuit diagram of the filter (see Fig.3).

This functional link is an active second-order filter built on the basis of an operational amplifier. Since in this work a tenth-order filter is calculated, it will be necessary to connect five second-order links. To do this, using the denormalized filter coefficients and specifying two element values for each second-order cascade (in our case, this is C1, C2), we calculate the remaining elements of the cascades from the following expressions. Therefore, we will set two elements arbitrarily. It is more rational to set the capacitances of the capacitors. However, these values must satisfy the following condition

$$C_2 > C_1 \cdot \left(\gamma \cdot \frac{C}{2\alpha} - 1\right)$$

3. Approximation Using Butterworth Polynomials

The approximation task is to synthesize a certain frequency function that satisfies the requirements for the amplitude-frequency characteristic or the characteristic of the working attenuation of the filter to be developed. It is highly convenient to represent the frequency function as a characteristic of the working attenuation, which is expressed by formula (1).

For the filtering function $\psi(\omega)$ included in this formula, it is recommended to have values close to zero in the passband and as large as possible in the stopband, while $\psi(\omega)$ in the general case it itself is a fractional function.

Where

$$\gamma = \alpha^2 + \beta^2$$

By analysing the expressions of the filter transfer characteristic, we shall determine the types of conductivities to ensure the required degree p. Thus, we can conclude that the conductivities Y1, Y2 and Y5 should be capacitors, and the conductivities Y3 and Y4 should be resistors.

$$Y_1 = pC_1, Y_4 = \frac{1}{R_4}, Y_3 = \frac{1}{R_3}, Y_2 = pC_2, Y_5 = pC_5 \tag{16}$$

Accordingly, the formula for the transfer characteristic of a bandpass filter for the case under consideration has the form

$$K(p) = \frac{p}{R_2C_3\,\frac{C_1+C_2}{C_1}\left(p^2+\frac{R_1+R_2}{R_1R_2(C_1+C_2)}p+\frac{1}{R_1R_2R_3(C_1+C_2)}\right)} \tag{17}$$

In addition, from the comparison of Eq.(15) and Eq.(17), we have

The filter function $\psi(\omega)$- can be also obtained from the coefficient of the filter transfer function through the following relationship

$$[K(p)]_{p=jw} = \frac{1}{\sqrt{1+\varepsilon^2\psi(\omega)}} \tag{19}$$

Here ε is the percentage of transmission non-uniformity.

In engineering practice, methods for obtaining the filtering function $\psi(\omega)$ and, consequently, the complex transfer function $K(j\omega)$ are conveniently classified by the frequency response approximation criterion:

1) equal-wave (uniformly oscillatory) approximation in the passband and in the stopband;

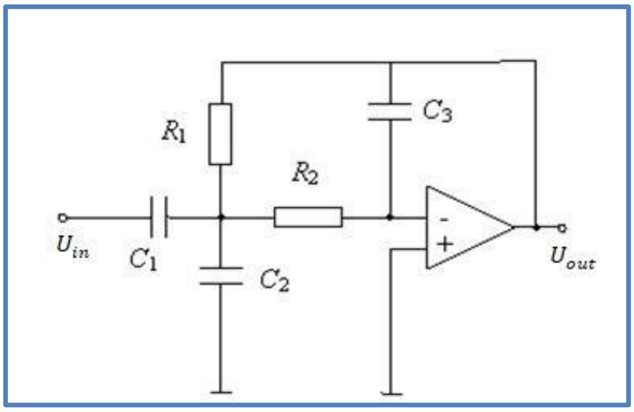

Fig. 3 Second-order bandpass filter element based on the Rauch structure with a capacitive voltage divider.

2) equal-wave approximation in the passband;

3) maximally flat approximation in the passband.

In the last two cases, the attenuation in the stopband increases monotonically with distance from the cutoff frequency. A fairly large number of types of polynomials and fractions can be used as a filtering function, but the most popular today are the Butterworth, Chebyshev and Bessel approximations.

4. Results and Discussion

A widely used method of approximating the idealized characteristic of a low-pass filter is to find the characteristic of the working attenuation with the maximally flat approximation. The filter function in this case is represented by Butterworth polynomials

$$\psi(\omega) = B_m(\Omega) = \Omega^m \tag{20}$$

Taking the last expression into account in (1), we can reach a Butterworth filter model of the following form

$$a(\omega) = 20lg\left(\frac{1}{|K(\omega)|}\right) = 10lg\ (1 + \varepsilon^2\Omega^m \tag{21}$$

The fundamental frequency and time characteristics of Butterworth filters of different degrees calculated based on the expressions we obtained above are given in Fig. 4-9.

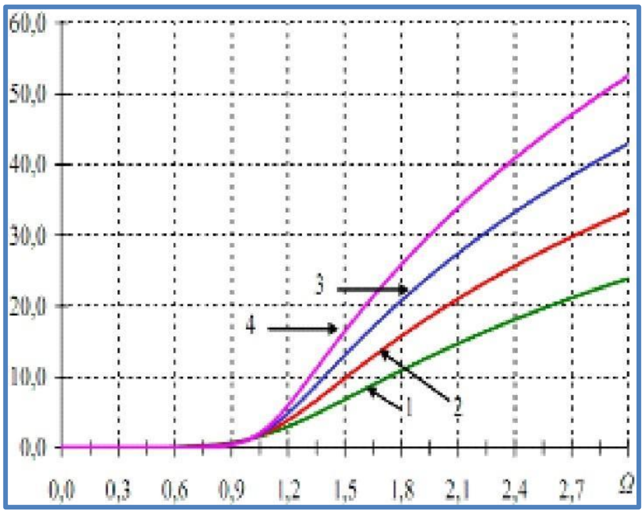

Fig. 4 Operating attenuation characteristics of Butterworth filters (OACB) of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6).

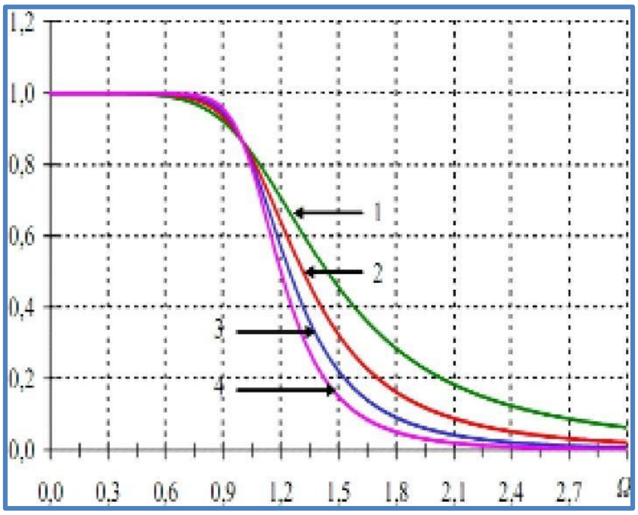

Fig. 5 Amplitude-frequency characteristics of Butterworth filters (AFCBF) of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6).

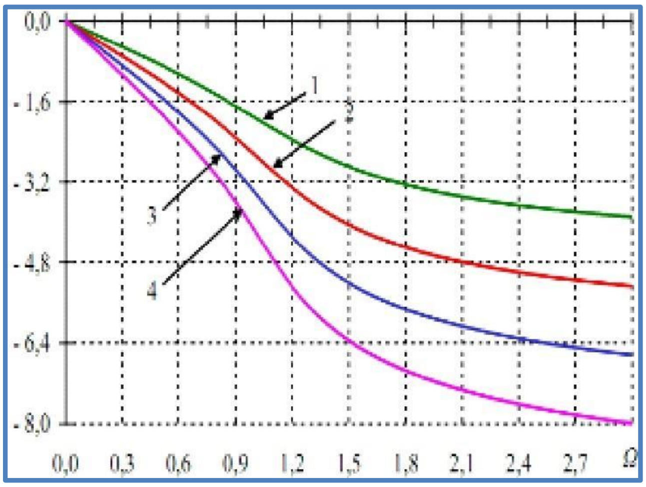

Fig. 6 Phase-frequency characteristics of Butterworth filters (PFCBF) of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6.

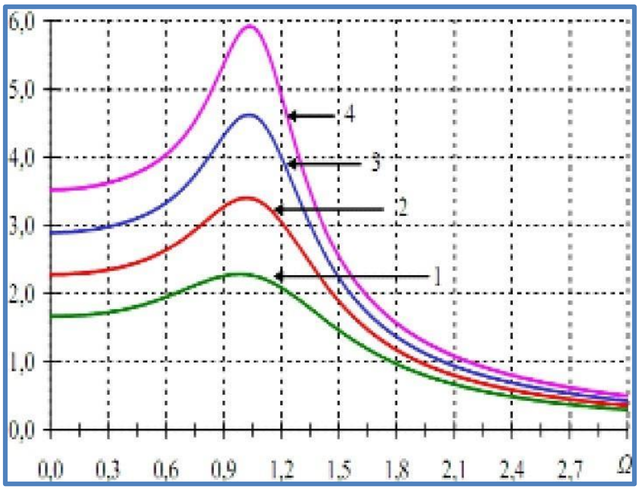

Fig. 7 Delay characteristics (DCBF) of Butterworth filters of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6).

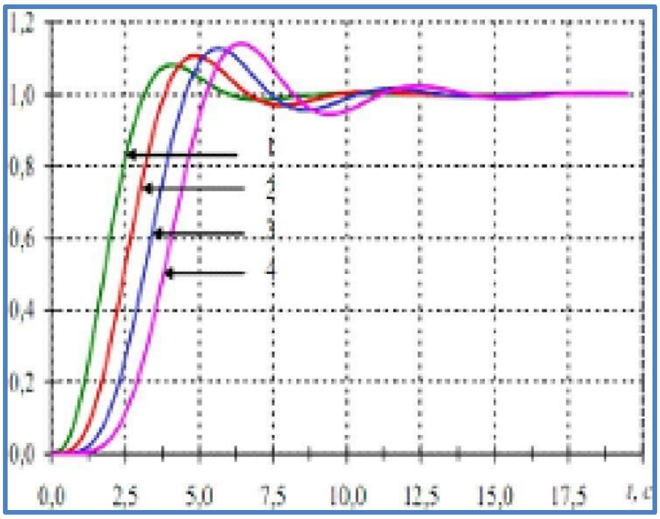

Fig. 8 Transient responses of Butterworth filters (TRBF) of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6).

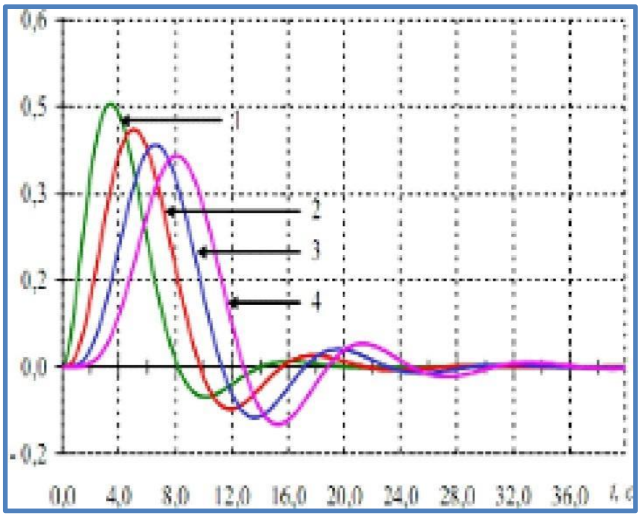

Fig. 9 Impulse responses of Butterworth filters (IRBF) of different orders (1-m=3; 2-m=4; 3-m=5; 4-m=6).

Thus, based on the analysis of the frequency characteristics of Butterworth filters, the following results were obtained.

5. Conclusion

In this study, a tenth-order bandpass filter with Butterworth approximation was developed. Five links of the second-order Rauch structure were used as circuit implementation.

The Operating Attenuation Characteristics have a monotonically increasing character in the passband (up to the cutoff frequency) and a monotonous character in the transition region and stopband. The Amplitude-Frequency Characteristics have a monotonically decreasing character in the passband (up to the cutoff frequency) and a monotonous character in the transition region and stopband.

The degree of approximation of the characteristics to the idealized ones increases along with the increasing order of the Butterworth polynomial (filter order). The characteristics at a frequency equal to zero have the same attenuation for even and odd orders.

The Phase-Frequency Characteristic deviates increasingly from linear along with increasing filter order, and its slope increases.

The analysis of the time characteristics indicates that both the duration of the transient process and the amplitude of oscillations increase along with increasing the filter order, whereas the swing of the main lobe of the characteristic decreases along with a simultaneous increase in the duration at the 0.5 level.

Declaration

Author Contribution: Conceive– H.A.; Design– H.A., S.A; Experimental Performance, Data Collection and Processing– H.A., S.A; Analysis and Interpretation– H.A., S.A., H.D.; Literature Review– H.A., S.A; Writer– H.A., H.D.; Critical Review– H.A., H.D.

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